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NOTE ON SIMILARITY CONDITIONS FOR FLOWS WITH HEAT TRANSFER

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SUMMARY

The conditions for similarity of flow for a compressible fluid with heat transfer are examined with particular reference to variation of stagnation temperature and scale of the model. Similar flows are possible only when certain conditions are fulfilled by the intrinsic properties of the gas. The restrictions on the characteristics of the free stream and on the scale of the model necessary to obtain similar flows are pointed out. For similar flows in air, the following expression gives the ratio of local rates of heat transfer on the model to those on the prototype:

$$\frac{H_{tunnel}}{H_{flight}} = \left(\frac{T_{o_{tunnel}}}{T_{o_{flight}}}\right)^{1.95} \frac{L_{flight}}{L_{tunnel}}$$

where

H heat transfer per unit area per unit time

T_o absolute stagnation temperature

L typical length

Important simplification of the experimental procedure for tests simulating flight at high Mach numbers is made possible by divorcing the fluid-heat-conduction problem from the heat-radiation problem and interpreting the wind-tunnel results by means of similarity relations developed in this paper.

INTRODUCTION

The conditions for similarity of flow between model and prototype are of basic importance in problems associated with model testing. The usual wind-tunnel test data are generally assumed to be strictly applicable to flight conditions when the values of the Reynolds number and Mach number are the same for the prototype in flight as for the model in the wind tunnel. Identity of the Reynolds number and Mach number ceases to be a sufficient condition for flow similarity if the effects of heat transfer are appreciable.

The effects of heat transfer on the flow pattern can be important at high Mach numbers. For example, calculations by Lees (reference 1) have indicated that heat transfer from the boundary layer to the body has a marked stabilizing effect on the laminar boundary layer. The stagnation temperature corresponding to a Mach number of 4.0 in air and a free-stream temperature of 400° F absolute is 1680° F absolute, and the body surface will tend to assume a temperature close to this value. At such high temperatures, however, the amount of heat dissipated per unit area from the body by radiation is large. Under steady conditions, a corresponding amount of heat must be transmitted to the body from the surrounding fluid. Wind-tunnel measurements of the drag of bodies at high Mach numbers should, therefore, be made with rates of heat transfer to the surface corresponding to flight conditions in order for the transition point to occur at the right place and, hence, for the skin friction to have the proper value.

Exact duplication of flight conditions in a wind tunnel is extremely difficult. Even if it were possible to build a supersonic wind tunnel which would produce an air stream large enough to take a full-size model and have a stagnation temperature equal to the expected flight value, the conditions for radiant-heat transfer would be greatly different from those in flight unless special care were taken to control the temperature of the tunnel walls.

Great simplification of experimental procedures would result if it were possible to interpret in terms of corresponding flight phenomena the results of tests run with considerably lower stagnation temperatures than those which occur in flight and with models of different size from the prototype. For such tests, the models would be refrigerated or otherwise maintained at a temperature considerably below the normal stagnation temperature in order that the test conditions would correspond to flight conditions with heat loss by radiation. The purpose of the present paper is to examine the similarity conditions for flows with heat transfer with particular reference to the effects of model size and stagnation temperature and to outline a possible experimental procedure for obtaining the basic heat-transfer information necessary to predict values of the surface temperature in flight.

SYMBOLS

F Froude number $\left(V^2/Lg\right)$

H heat transfer per unit area per unit time

L	typical length
M	Mach number (V/a)
Nu	Nusselt number (HL/kT)
Pr	Prandtl number $(c_p\mu/k)$
R	gas constant $(p/\rho T)$
Re	Reynolds number $(\rho VL/\mu)$
T	absolute temperature
V	typical velocity
a	velocity of sound
c_p	specific heat at constant pressure
c_v	specific heat at constant volume
g	acceleration due to gravity
k	heat conductivity (gas property), that is, heat transfer per unit time per unit area per unit temperature gradient (fluid at rest)
p	pressure
u, v, w	components of velocity in rectangular coordinates
x, y, z	rectangular coordinates
β	temperature coefficient of volume expansion
γ	ratio of specific heat at constant pressure to specific heat at constant volume
μ	coefficient of viscosity
ρ	density

Subscript:

o	stagnation or reference conditions

ANALYSIS

In examining the possibility of simulating flight conditions in a wind tunnel, it is of some interest to determine the effects of variations in the stagnation temperature and scale of the model on the rates of heat transfer over the model necessary to maintain similar fields of flow, in particular, similar boundary layers and similar temperature distributions within the boundary layers. The approach to the determination of conditions for flow similarity is similar to that of reference 2. It is assumed that the following conditions completely determine the field of flow about a body: (a) shape of the body, (b) distribution of local rates of heat transfer per unit area over the surface of the body, (c) physical properties and conditions of the fluid, (d) field of flow in which the body is placed, and (e) gravitational or other force field in which the phenomena take place. The kinds of quantities necessary for specifying these conditions are as follows:

L length

V velocity

T absolute temperature

H heat transfer per unit area per unit time

k absolute heat conductivity

μ absolute viscosity

c_p specific heat at constant pressure

ρ mass density

p pressure

β temperature coefficient of volume expansion

g acceleration due to gravity

The number of kinds of variables required to specify the problem is thus 11. Since there are four fundamental types of units - mass, length, time, and temperature - by the Π theorem given in reference 2, seven independent nondimensional combinations of the different variables are required to specify the problem.

The general problem of the flow of a compressible fluid with heat transfer can, therefore, be formulated symbolically in the following nondimensional form:

Pr. No Froude No Reynolds Nusselt

$$f\left[\beta T,\ \frac{kT}{\mu V^2},\ \frac{c_p\mu}{k},\ V\sqrt{\frac{\rho\left(c_p - \frac{p}{\rho T}\right)}{pc_p}},\ \frac{V^2}{Lg},\ \frac{\rho VL}{\mu},\ \frac{HL}{kT}\right] = 0 \tag{1}$$

Most of the nondimensional parameters occurring in equation (1) are commonly referred to by definite names as follows:

$$\frac{c_p\mu}{k} = \text{Prandtl number}$$

$$\frac{V^2}{Lg} = \text{Froude number}$$

$$\frac{\rho VL}{\mu} = \text{Reynolds number}$$

$$\frac{HL}{kT} = \text{Nusselt number}$$

If the fluid is assumed to obey the gas law, $\frac{p}{\rho T} = \text{Constant}$,

$$V\sqrt{\frac{\rho\left(c_p - \frac{p}{\rho T}\right)}{pc_p}} = V\sqrt{\frac{\rho}{\gamma p}} = \frac{V}{a} = M$$

and, also,

$$\beta T = 1$$

Equation (1) may then be written

$$f\left(\frac{kT}{\mu V^2},\quad Pr,\quad M,\quad F,\quad Re,\quad Nu\right) = 0 \tag{2}$$

In accordance with the definition of similar flows, all the local values of the parameters occurring in equation (2), as well as more directly descriptive ratios such as u/V, v/V, w/V, p/p_o, and ρ/ρ_o, must have the same values at corresponding points x/L, y/L, and z/L. If flow patterns are to remain similar as the stagnation temperature T_o and the size of the model L are varied, certain conditions must be fulfilled by the gas and the local rates of heat transfer over the body must vary in a specific manner. These conditions are derived from consideration of the various parameters occurring in equation (2).

Condition 1 - the parameter $kT/\mu V^2$: Since T is proportional to the local value of a^2 if γ is assumed constant, T/V^2 is proportional to $1/M^2$ and, hence, will be the same at corresponding points. The value of k/μ must therefore be independent of the temperature.

Condition 2 - the parameter Pr: With the previous condition 1 that k/μ must be constant, constancy of the Prandtl number Pr demands that c_p be independent of the temperature. This condition is consistent with the previous assumption that γ be constant.

Condition 3 - the parameter M: The value of M will be the same at corresponding points, provided that the free-stream velocity is varied as the square root of the temperature; that is, V must be proportional to $\sqrt{T_o}$.

Condition 4 - the parameter F: Since, from condition 3, V must be proportional to $\sqrt{T_o}$, in order for the value of V^2/Lg to remain unchanged, L must be proportional to T_o (g considered constant).

Condition 5 - the parameter Re: Since the velocity must vary as the square root of the temperature, constancy of the Reynolds number demands that μ/ρ be proportional to $L\sqrt{T}$ or $L\sqrt{T_o}$.

Condition 6 - the parameter Nu: In order to maintain similar flow as L and T_o are varied, the local rates of heat transfer per unit area from the surface H must be proportional to kT/L.

Condition 1 and condition 2, as well as the general condition that $\frac{p}{\rho T}$ = Constant, depend on intrinsic properties of the gas. The kinetic theory of gases composed of small hard spheres (reference 3) indicates that $p/\rho T$, k/μ, and c_p should be constant. The type of gas assumed in this elementary theory, therefore, satisfies the necessary conditions for flow similarity. In many respects, air behaves substantially as such a gas. For example, $p/\rho T$ is found to be substantially constant over a range of temperatures from slightly above those required for liquefaction to those at which the effects of dissociation begin to be appreciable, if the pressure is not too high. On the other hand, the experimentally determined variation of c_p, μ, and k with temperature is somewhat different from that predicted from the elementary theory.

The data of reference 4 indicate a 25-percent increase in the ratio k/μ over the range of temperatures from -100° F to 1600° F. Variation of k/μ affects the value of $kT/\mu V^2$, the index of the ratio of the flow of heat to the energy dissipated by viscous friction. Little is known of the effect of variations of this parameter on either the heat-flow pattern or the kinematic-flow pattern. It seems probable, however, that the primary effect of the parameter $kT/\mu V^2$ is on the value of the surface temperature corresponding to zero heat flow to or from the fluid at the boundary (equilibrium temperature). Large values of k/μ would correspond to low equilibrium temperatures. It has generally been observed in heat-transfer investigations that data for widely different values of k/μ correlate well on the basis of the Prandtl number. In the absence of more detailed information, it seems reasonable to assume that the changes in flow phenomena associated with variations of k/μ will be of the same order of magnitude as the variations caused by corresponding changes in the Prandtl number.

The data of reference 4 indicate that the Prandtl number for air varies from 0.743 at -100° F to 0.643 at 1600° F. Thus, the variation of c_p with temperature partly compensates the variation of k/μ. The effect on the flow pattern of a 15-percent variation of the Prandtl number is estimated to be small since the Prandtl number apparently occurs only to the 0.4 or 0.3 power in the relations for heat transfer given in reference 5. Some of this variation of the Prandtl number may be caused by inaccuracies in the measured values of k. The probable error in k is estimated in reference 4 to be 7 percent.

In addition to the effect of a variation of c_p on the Prandtl number, variation of c_p is always associated with a variation in the ratio of specific heats γ. Variations of γ cause variations in the pressure ratios, density ratios, and so forth, associated with given values of the Mach number and, therefore, can have a marked effect on the flow pattern. The variations of γ become progressively more important above temperatures of about 700° F absolute as seen in figure 1. The data presented in figure 1 were derived from values c_p/R presented in reference 6 and the relation $c_p - c_v = R$.

The preceding discussion indicates the extent to which air may be assumed to act as a perfect gas for the purposes of this analysis. Condition 1 and condition 2 are substantially satisfied by air, provided that the actual air temperature in the field of flow does not vary sufficiently for the corresponding variations of k/μ and γ to have an appreciable effect. Variations of k/μ and γ will limit the range of values of T_o over which strict similarity is obtainable. The amount of variation in these ratios that is permissible will depend in large measure on the types of phenomena that are of primary interest; that is, for example, whether heat transfer, boundary-layer stability, or force coefficients are the phenomena under investigation.

It may be noted that the effects of variations of γ are largest in the field of flow outside the boundary layers. The pressures about efficient supersonic shapes with attached shock waves ordinarily do not approach the stagnation pressure very closely. The variation of γ with temperature is not expected to have a serious effect on flow similarity unless the temperature outside the boundary layer is somewhat higher than 700° F absolute.

Conditions 3 to 6 affect mainly the free-stream and boundary conditions. As stated previously, if condition 3 and condition 4 are to be satisfied simultaneously and if the acceleration due to gravity g is considered constant, L and T_o can no longer be varied independently but L must be proportional to T_o. In most cases of high-speed flows, the effects of free-convection currents of which the Froude number is an index are unimportant. Under these circumstances, condition 4 may usually be neglected and L and T_o may be varied independently. Condition 6 can be interpreted as specifying the necessary local rates of heat transfer on the model. The variation of k with temperature, taken from reference 7 (the basis for the values presented in reference 4), is given in figure 2. It is seen that k varies substantially as a power of the absolute temperature; that is, $\frac{k}{k_o} = \left(\frac{T}{T_o}\right)^n$. Since T/T_o is the same at corresponding points for similar flows, the conductivity at all points in the field of flow is proportional to the conductivity at stagnation conditions. Condition 6 then reduces to the form

$$H \propto \frac{k_o T_o}{L}$$

From figure 2 it is seen that k is approximately proportional to $T^{0.95}$. Hence, condition 6 may be stated

$$\frac{H_{tunnel}}{H_{flight}} = \left(\frac{T_{o_{tunnel}}}{T_{o_{flight}}}\right)^{1.95} \frac{L_{flight}}{L_{tunnel}} \tag{3}$$

APPLICATION TO WIND-TUNNEL TESTING

The preceding analysis shows the necessary conditions that must be fulfilled by the free stream and at the surface of the model in order for the flow about the model to be similar to that occurring in flight. In most supersonic wind tunnels the absolute stagnation temperature is approximately equal to the ambient room temperature; whereas in flight the stagnation temperature tends to be high for the higher Mach numbers. The actual temperature assumed by the surface of the body in flight is the result of a heat balance between conduction through the boundary layer to the body and heat dissipation from the body by radiation.

Simultaneous simulation of both fluid-heat-transfer and radiant-heat-transfer conditions in a wind tunnel is extremely difficult. In addition to obtaining stagnation temperatures equal to the flight values and cooling the tunnel walls in order to simulate flight-radiation conditions, a brief check of the similarity conditions shows that it is also necessary for the linear dimensions of the model to be the same as those of the prototype.

A much simpler procedure for simulating flight conditions in a wind tunnel is obtained by divorcing completely the problem of heat transmission by radiation from that of heat transmission by surface conduction and then by using the similarity relations given in the preceding section to set up convenient tunnel operating conditions. This simplified approach may be illustrated by the following example.

Consider the case of an airfoil section having sufficient internal heat conductivity so that the surface temperature can be considered uniform. It is desired to find the characteristics of this airfoil corresponding to flight conditions at a definite value of the Mach number and Reynolds number. The stagnation temperature in flight is assumed to be known. The wind-tunnel air stream is adjusted so as to give the flight value of the Reynolds number and Mach number for a model of convenient size. The stagnation temperature of the tunnel air stream is chosen sufficiently low so that all heat transfer from the model by radiation is negligible. (If the stagnation temperature of the tunnel is equal to the ambient temperature in flight, the radiant-heat transfer per unit area in the tunnel will be approximately $\frac{1}{300}$-th that in flight at a Mach number of 4.0.) The characteristics of the airfoil and the surface temperature corresponding to a range of heat-transfer rates from the fluid to the model (various degrees of refrigeration of the model) are measured in the tunnel.

The relation between the heat transfer by radiation and the surface temperature for flight conditions can be calculated from the Stefan-Boltzman law so that a curve of radiant-heat transfer from the surface against surface temperature can be plotted. By the use of equation (3) and the wind-tunnel heat-transfer data, a curve of heat transfer to the airfoil plotted against surface temperature is drawn for flight conditions. The surface temperature corresponding to the intersection of these two curves is the surface temperature that will occur in flight; and the aerodynamic characteristics of the model, measured in the tunnel at the corresponding ratio of surface to stagnation temperatures, are the same as those occurring under the specified conditions in flight, provided that the temperatures in the field of flow are not so high that variations of k/μ and γ are important.

When conditions are such that the temperatures over the surface of the body in flight cannot be assumed uniform, the problem of finding the distribution of surface temperatures in flight is much more complex. Nevertheless, since the laws for heat transmission by solid conduction and radiation are relatively simple, it seems probable that a convergent process of successive approximations to the surface temperature distributions can be devised.

CONCLUSIONS

Examination of similarity conditions for flows of a compressible fluid with heat transfer, with particular reference to variation of stagnation temperature and scale of the model, indicates the following conclusions:

1. In order for similar flows to be possible at various values of the stagnation temperature and for models of various scales, the gas must fulfill the following conditions:

The ratio of the heat conductivity to the viscosity must be independent of the temperature, or at least must have the same value at corresponding points for model and flight conditions.

The specific heat must be constant, or at least must have the same value at corresponding points.

The temperature coefficient of volume expansion must be inversely proportional to the absolute temperature.

2. Maintenance of corresponding local values of the flow parameters, Reynolds number, Mach number, Prandtl number, Froude number, and so forth, with variations of stagnation temperature and model scale imposes the following restrictions:

The free-stream velocity must be proportional to the square root of the stagnation temperature.

The kinematic viscosity must be proportional to the product of the model scale and the square root of the absolute stagnation temperature.

If gravity effects are important, the scale of the model must be proportional to the absolute stagnation temperature.

3. The range of stagnation temperatures over which similar flows are possible for air is limited by variations with temperature of k/μ, the ratio of heat conductivity to viscosity, and of γ, the ratio of specific heat at constant pressure to that at constant volume. Variations of γ are expected to have an important effect on flow similarity only if the temperature outside the boundary layer exceeds 700° F absolute.

4. Maintenance of corresponding temperature distributions over the surface demands that the following relation between the local rates of heat transfer at the surface for air be satisfied:

$$\frac{H_{tunnel}}{H_{flight}} = \left(\frac{T_{o_{tunnel}}}{T_{o_{flight}}}\right)^{1.95} \frac{L_{flight}}{L_{tunnel}}$$

where

H	heat transfer per unit time per unit area
T_o	stagnation temperature
L	typical length

5. Subject to the limitations of conclusion 3, important simplifications of the experimental procedure for tests simulating flight at high Mach numbers are made possible by divorcing the fluid-heat-conduction problem from the heat-radiation problem and interpreting the wind-tunnel results by means of the foregoing similarity relations.

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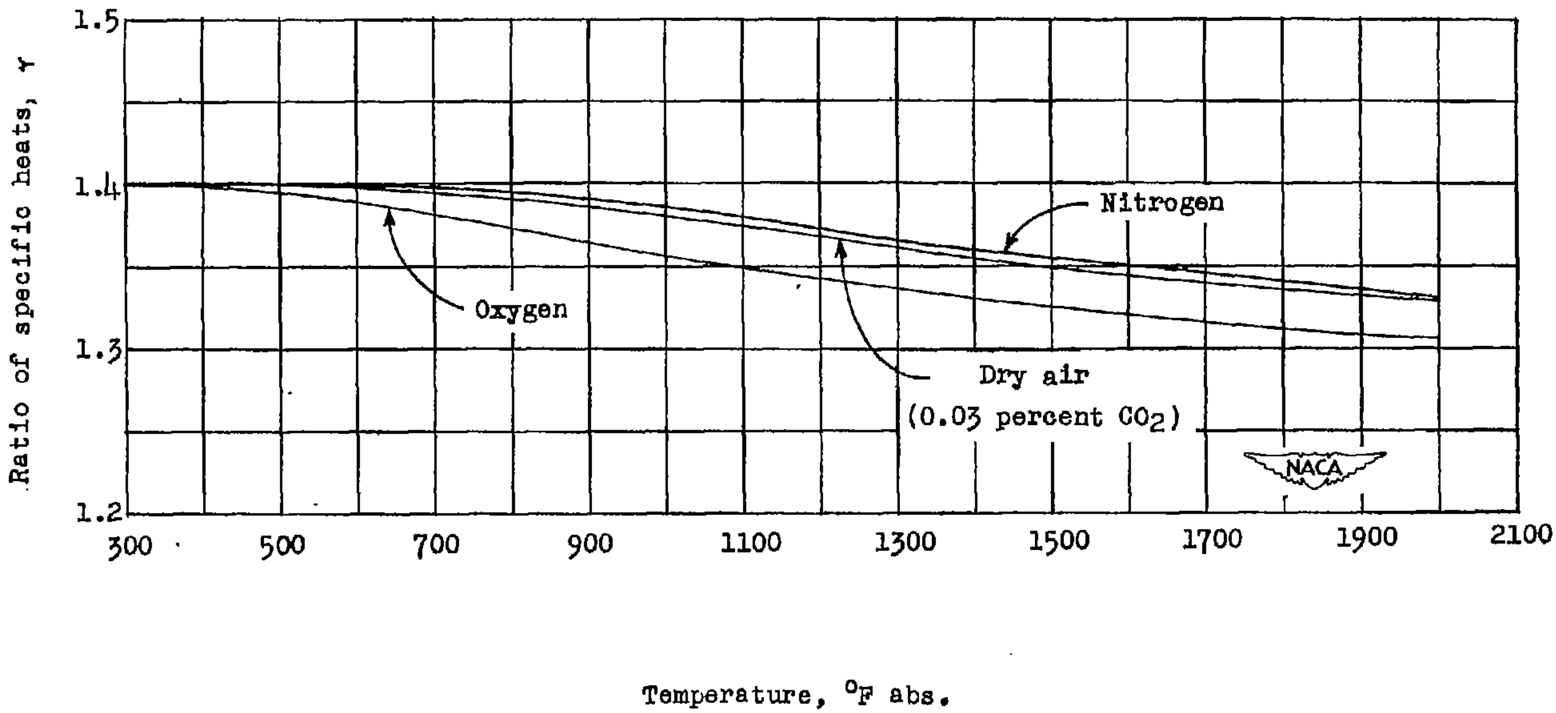

Figure 1.- Variation of ratio of specific heats γ with absolute temperature, extrapolated to zero pressure, calculated from the data and methods of reference 6.

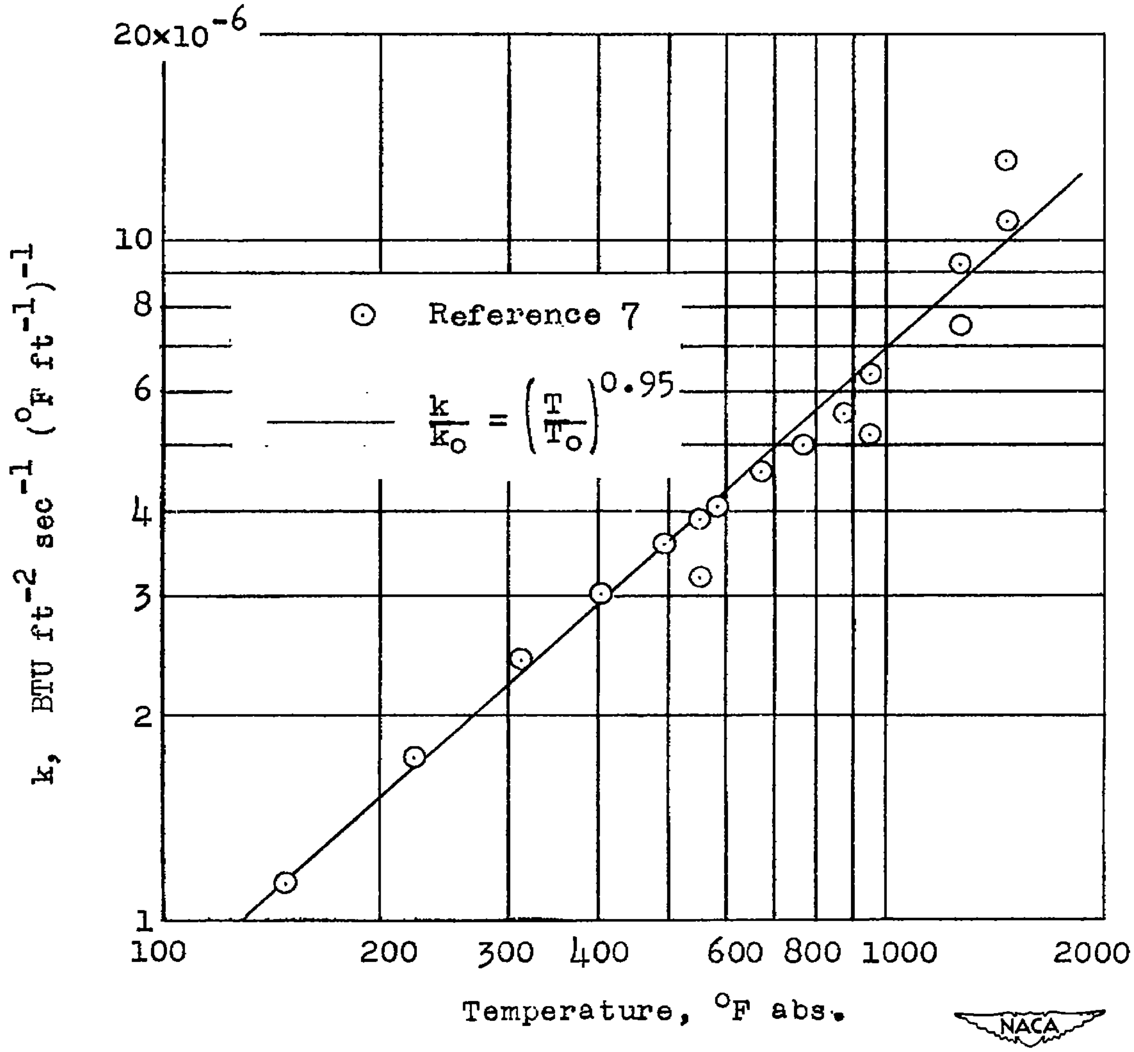

Figure 2.- Variation of absolute heat conductivity with absolute temperature for air.